

Lecture 20 CH102 A3 (TuTh 5 pm)

Tuesday, April 5, 2016

For today ...

- Continue Ch. 17 Spontaneous change – how far?
 - Heat flow and entropy change
 - Spontaneity of phase transitions
 - The role of ΔS in colligative properties

Next lecture:

- Continue Ch. 17 Spontaneous change – how far?
 - Entropy change for chemical reactions



Lecture 20 CH102 A3 (TuTh 5 pm) Spring 2016

Heat (energy) flow entropy change

We can answer these questions using by analyzing **energy dispersal** and the corresponding **entropy change**.

Which energy change (q, m) $(q + 1, m)$ has **larger ΔS** ...

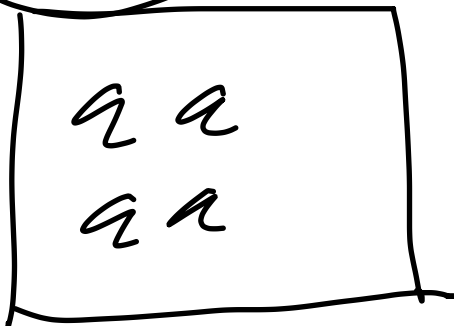
adding 1 quantum to 4: $(4, m)$ $(5, m)$ or ...

adding 1 quantum to 9: $(9, m)$ $(10, m)$?

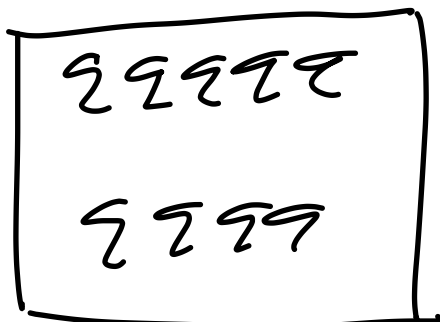
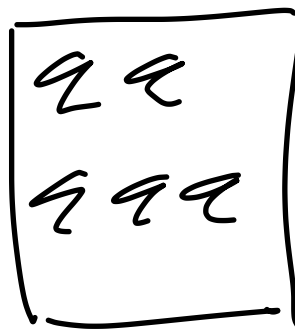
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4 molecules

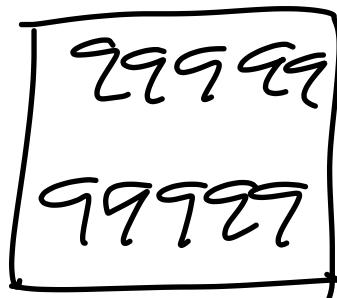
4
LARGER ΔS ??



+ 1 q



+ 1 q



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[TP] Evaluate $W_e(4 \text{ quanta}, 4 \text{ molecules})$.

25% 1. $14 \times 5 = 70$

25% 2. $7 \times 5 = 35$

25% 3. $4 \times 5 = 20$

25% 4. None of the above

$$W_e(4, 4)$$

$$= \frac{(4 + (4-1))!}{4! (4-1)!}$$

$$= \frac{7!}{4! 3!}$$

$$= \frac{7 \times \cancel{6} \times 5}{\cancel{3} \cdot \cancel{2}} = 35$$

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5

1

$$W_e(5, 4) = \frac{(5+3)!}{5! 3!} = \frac{8 \cdot 7 \cdot 6}{6} = 8$$

[TP] Evaluate $W_e(9 \text{ quanta}, 4 \text{ molecules})$.

25% 1. $11 \times 10 = 110$

25% 2. $10 \times 22 = 220$

25% 3. $13 \times 11 = 143$

25% 4. None of the above

$$\boxed{W_e(9,4)} = \frac{(9+3)!}{9!3!} = \frac{\overset{2}{12} \cdot 11 \cdot 10}{\cancel{3} \cdot \cancel{2}}$$
$$= 22 \cdot 10 = 220$$

[TP] Evaluate $W_e(10 \text{ quanta}, 4 \text{ molecules})$.

25% 1. $13 \times 12 \times 11 = 1716$

25% 2. $12 \times 11 = 132$

25% 3. $13 \times 2 \times 11 = 286$

25% 4. None of the above

$$W_e(10, 4) = \frac{(10+3)!}{10! 3!} = \frac{13 \cdot \cancel{12} \cdot 11}{\cancel{6}}$$
$$= 13 \cdot 2 \cdot 11 = 286$$

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[TP] The general expression for the entropy change of a 4 molecule system with q quanta **gaining** 1 quantum is ...

- 25% 1. $W_e(q + 1, 4) - W_e(q, 4)$
 25% 2. $\ln[W_e(q + 1, 4)] - \ln[W_e(q, 4)]$
 25% 3. $\ln[W_e(q + 1, 4) / W_e(q, 4)]$
 25% 4. Both 2 and 3

$$S = k_B \ln \omega$$

$$\Delta S = k_B \ln \left(\frac{\omega_f}{\omega_i} \right) = k_B [\ln \omega_f - \ln \omega_i]$$

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5

8

$$\omega_f = W_e(q+1, 4)$$

$$\omega_i = W_e(q, 4)$$

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[TP] Which is larger?

- 33% 1. $W_e(10, 4) / W_e(9, 4)$
 33% 2. $W_e(5, 4) / W_e(4, 4)$
 33% 3. More information is needed

$$\frac{W_e(10, 4)}{W_e(9, 4)} = \frac{286}{220} = 1.3$$

$$\frac{W_e(5, 4)}{W_e(4, 4)} = \frac{56}{35} = 1.6$$

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5

9

$$W_e(5, 4) = \frac{(5+3)!}{5!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 4}{3 \cdot 2 \cdot 1} = 56$$

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Heat (energy) flow entropy change

Which change (q, m) $(q + 1, m)$ has **larger ΔS** ,


adding 1 to 5: $(4, m)$ $(5, m)$ or ...

adding 1 to 9: $(9, m)$ $(10, m)$?

$(4, m)$ $(5, m)$ has **larger ΔS** than $(9, m)$ $(10, m)$

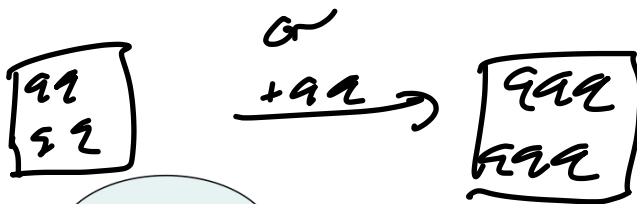
This result illustrates that ...

ΔS is larger, the lower T

 10

[TP] Which is larger?

- 33% 1. $W_e(6, 4) / W_e(4, 4)$
- 33% 2. $W_e(5, 4) / W_e(4, 4)$
- 33% 3. More information is needed



Response Counter

5

11

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Heat (energy) flow entropy change

Which change (q, m) $(q + 1, m)$ or $(q + 2, m)$ has **larger ΔS** ,

adding 1 to 4: $(4, m)$ $(5, m)$ or ...

adding 2 to 4: $(4, m)$ $(6, m)$?

$(4, m)$ $(6, m)$ has **larger ΔS** than $(4, m)$ $(5, m)$

This result illustrates that ...

ΔS larger, the greater ΔH



12

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Heat (energy) flow entropy change


Which changes have **larger ΔS** ?

(4, m) (5, m) has **larger ΔS** than (9, m) (10, m)

(4, m) (6, m) has **larger ΔS** than (4, m) (5, m)

These results illustrate that ...

$$\Delta S = \Delta H / T$$

 13


$$\Delta S \propto \frac{1}{T}$$

$$\Delta S \propto \Delta H$$

$$\Rightarrow \Delta S = \frac{\Delta H}{T}$$

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Spontaneity of phase transitions



14

$\Delta S_{\text{universe}}$

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Taking stock

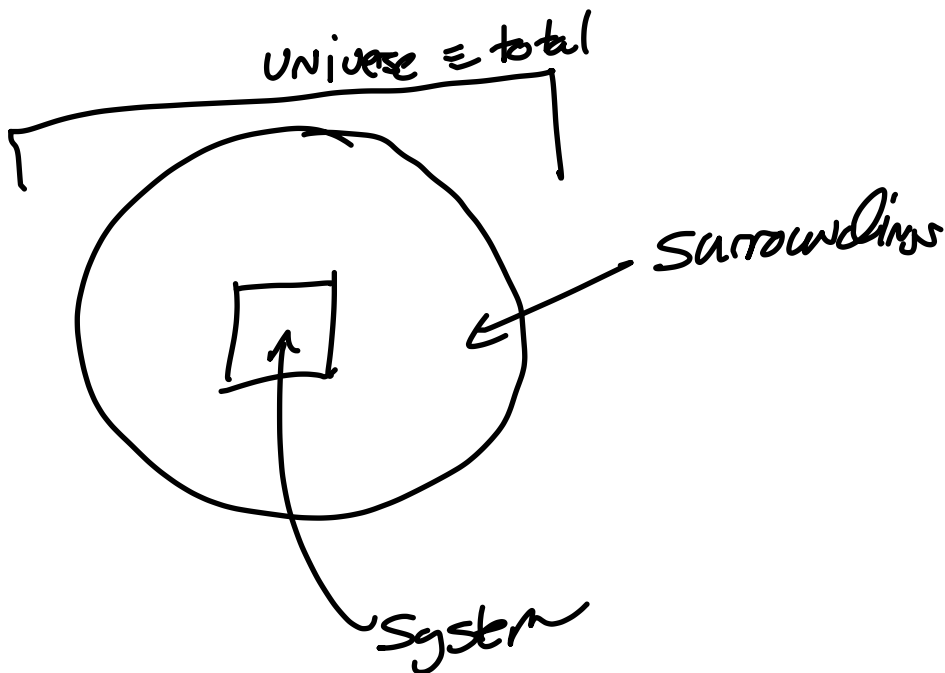
Spontaneity **means** that ...
 $\Delta S_{\text{tot}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} > 0$

Spontaneity **does not** require that ...
 $\Delta S_{\text{sys}} > 0$ or $\Delta S_{\text{surr}} > 0$

A neat illustration of the **separate roles** of ΔS_{sys} and ΔS_{surr} is understanding why **steam condenses** and **water boils**

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15



$$\Delta S_{sur} = \frac{\Delta H_{sur}}{T}$$

Where does this come from?
heat transfer to or from system!

steam [?] water at 94 °C

Super cooled steam at 94 °C condenses **spontaneously** to water.

Spontaneity means $\Delta S_{tot} > 0$

But "gas [?] liquid" means $\Delta S_{sys} < 0$

This means it must be ΔS_{sur} that makes $\Delta S_{tot} > 0$

How to get ΔS_{sur} ?

The trick: $\Delta S_{sur} = \Delta H_{sur} / T = -\Delta H_{sys} / T$

Hence we can always write $\Delta S_{tot} = \Delta S_{sur} + \Delta S_{sys}$ as

$$\Delta S_{tot} = -(\Delta H_{sys} / T) + \Delta S_{sys}$$

SYSTEM: exothermic
SURROUNDINGS: endothermic

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Gas
lots of arrangements

liquid
fewer arrangements

$\Delta S_{tot} = \Delta S_{sys} + \Delta S_{sur}$

$\Delta S_{sys} \ominus$

$\Delta S_{sur} \oplus$ **MUST BE!!**

$$\Delta H_{\text{sys}} = \overbrace{\Delta H_{\text{CONDENSATION}}}^{(-)} = - \overbrace{\Delta H_{\text{VAP}}}^{(+)}$$

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steam [?] water at 94 °C

$\Delta S_{\text{tot}} = -\Delta H_{\text{sys}} / T + \Delta S_{\text{sys}}$

At **100 °C**, steam and water are in equilibrium, so ...

$\Delta S_{\text{tot}} = 0 = +\Delta H_{\text{vap}} / (373 \text{ K}) + \Delta S_{\text{sys}}$

From this we know that ΔS_{sys} ...

$= -\Delta H_{\text{vap}} / (373 \text{ K})$

$= -(40.65 \times 10^3 \text{ J/mol}) / (373 \text{ K}) = -108.9 \text{ J/(mol K)}$

For **other temperatures** ΔS_{tot} ...

$= + (40.65 \times 10^3 \text{ J/mol}) / T - 108.9 \text{ J/(mol K)}$

ΔH for process

ΔS_{sys}

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17

$$-\Delta H_{\text{sys}} = -(-\Delta H_{\text{vap}})$$

$$-\Delta H_{\text{sys}} = +\Delta H_{\text{vap}}$$



endothermic process $\Delta H (+)$
(add heat)



exothermic process $\Delta H (-)$
(give off heat)

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[TP] For steam water

$$\Delta S_{\text{tot}} = + (40.65 \times 10^3 \text{ J/mol})/T - 108.9 \text{ J/(mol K)}$$

At $T = 94^\circ\text{C}$, ΔS_{tot} evaluates to ...33% 1. < 0 33% 2. $= 0$ 33% 3. > 0

ΔT 100°C , $\Delta S_{\text{tot}} = 0$

ΔT $< 100^\circ\text{C}$, T smaller, so first term bigger, $\Delta S = (+)$

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5

18

spontaneous -

yes, STEAM
condenses below
 100°C !

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[TP] For steam [?] water

$$\Delta S_{\text{tot}} = + (40.65 \text{ J/mol})/T - 108.9 \text{ J/(mol K)}$$

At $T = 100 \text{ }^\circ\text{C}$, ΔS_{tot} evaluates to ...0% 1. < 0 0% 2. $= 0$ 0% 3. > 0

100 °C is the equilibrium temperature!

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20

$\Delta S_{\text{total}} > 0$ SPONTANEOUS

$\Delta S_{\text{total}} = 0$ equilibrium

$\Delta S_{\text{total}} < 0$ not spontaneous

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[Quiz] For steam [?] water

$$\Delta S_{\text{tot}} = + (40.65 \text{ J/mol})/T - 108.9 \text{ J/(mol K)}$$

At $T = 106 \text{ }^\circ\text{C}$, ΔS_{tot} evaluates to ...

0% 1. < 0

0% 2. $= 0$

0% 3. > 0

CONDENSATION OF STEAM
IS NOT SPONTANEOUS
AT TEMPERATURE
 $> 100 \text{ }^\circ\text{C}$!



Response
Counter

5

21